Numere reale

Numere scrise în baza 10

$\overline{abcd} = a \cdot 10^3 + b \cdot 10^2 + c \cdot 10 + d$, a,b,c,d cifre $\in \{0,1,2,3,4,5,6,7,8,9\}$, $a \neq 0$;

$\overline{a,bcde} = a + b \cdot 10^{-1} + c \cdot 10^{-2} + d \cdot 10^{-3} + e^{-4}$, a,b,c,d,e cifre **;**

Fracții

- Fracții zecimale finite: $\overline{a,b} = \frac{ab}{10}$; $\overline{a,bc} = \frac{abc}{100}$.
- Fracții zecimale periodice
  - simple $\overline{a,(b)} = \frac{\overline{ab} - a}{9}$; $\overline{a,(bc)} = \frac{\overline{abc} - a}{99}$
  - mixte $\overline{a,b(c)} = \frac{\overline{abc} - \overline{ab}}{90}$; $\overline{a,b(cd)} = \frac{\overline{abcd} - \overline{ab}}{990}$.

Puteri naturale ale numerelor reale

1. $(+a)^n = +a^n$

2. $(-a)^{2n} = +a^{2n}$

3. $(-a)^{2n+1} = -a^{2n+1}$

4. $a^m \cdot a^n = a^{m+n}$

5. $a^m : a^n = a^{m-n}$, $a \neq 0$

6. $a^m \cdot b^m = (a \cdot b)^m$

7. $a^m : b^m = \left(\frac{a}{b}\right)^m$, $b \neq 0$;

8. $\frac{1}{a^m} = \left(\frac{1}{a}\right)^m = a^{-m}$, $a \neq 0$;

9. $(a^m)^n = a^{mn} = (a^n)^m$;

10. $a^0 = 1$, $a \neq 0$;

11. $0^n = 0$, $n \neq 0$, $n \in \mathbf{N}$.

Puterile numerelor reale se extind atât pentru exponenţi raţionali pozitivi sau negativi, cât şi pentru exponenţi reali, puterile reale fiind definite cu ajutorul şirurilor de puteri raţionale. Aceste puteri au proprietăţi identice cu exponenţi numere naturale.

Identităţi fundamentale

Oricare ar fi $x,y,z,t,a,b,c \in \mathbf{R}$ şi $n \in \mathbf{N}$, avem:

1. $a^2 - b^2 = (a - b)(a + b)$;
2. $(a + b)^2 = a^2 + 2ab + b^2$;
3. $(a - b)^2 = a^2 - 2ab + b^2$;
4. $4ab = (a + b)^2 - (a - b)^2$;
5. $(a^2 + b^2)(x^2 + y^2) = (ax - by)^2 + (ax + bx)^2$;
6. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$;
7. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$;
8. $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - xz - yz)$;
9. $x^3 + y^3 + z^3 = (x + y + z)^3 - 3(x + y)(y + z)(z + x)$;
10. $a^4 - b^4 = (a - b)(a + b)(a^2 + b^2)$;
11. $a^4 + b^4 = (a^2 + b^2)^2 - a^2b^2$;
12. $a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$;
13. $a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$;

14. $(1 + a)(1 + a^2 + a^4) = 1 + a + a^2 + a^3 + a^4 + a^5$;
15. $a^6 + b^6 = (a^3 - 2ab^2)^2 + (b^3 - 2a^2b)^2$ (G. de Recquigny-Adanson);
16. $a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + \ldots + ab^{n-2} + b^{n-1})$;
17. $a^{2n} - b^{2n} = (a^2 - b^2)(a^{2n-2} + a^{2n-4}b^2 + \ldots + a^2b^{2n-4} + b^{2n-2})$;
18. $a^{2n+1} + b^{2n+1} = (a + b)(a^{2n} + a^{2n-1}b + \ldots + ab^{2n-1} + b^{2n})$;
19. $(1 + a + a^2 + \ldots + a^n)(1 + a^{n+1}) = 1 + a + a^2 + \ldots + a^{2n+1}$.

Radicali. Proprietãţi

1. $\sqrt[m]{a} = a^{\frac{1}{m}},\ a > 0$;
2. $\sqrt[m]{\frac{1}{a}} = \frac{1}{\sqrt[m]{a}} = a^{-\frac{1}{m}},\ a > 0$;
3. $\left(\sqrt[m]{a}\right)^m = a,\ a \geq 0$;
4. $\sqrt[m]{a} \cdot \sqrt[m]{b} = \sqrt[m]{ab},\ a,b \geq 0$;
5. $\left(\sqrt[m]{\frac{1}{a}}\right)^m = \frac{1}{a},\ a > 0$;
6. $\sqrt[m]{a} \cdot \sqrt[m]{b} \cdot \sqrt[m]{c} = \sqrt[m]{abc},\ a,b,c, \geq 0$;
7. $\sqrt[m]{a} : \sqrt[m]{b} = \sqrt[m]{\frac{a}{b}},\ a \geq 0,\ b > 0$;
8. $\sqrt[m]{a} \cdot \sqrt[n]{a} = \sqrt[m \cdot n]{a^{m+n}},\ a \geq 0$;
9. $\sqrt[m]{a} : \sqrt[n]{a} = \sqrt[m \cdot n]{a^{n-m}},\ a > 0$;
10. $\sqrt[n]{a^{nm}} = a^m,\ a \geq 0$;
11. $\sqrt[m]{a^n} = \left(\sqrt[m]{a}\right)^n = a^{\frac{n}{m}},\ a \geq 0$;
12. $\sqrt[mn]{a^{mp}} = \sqrt[n]{a^p},\ a > 0$;
13. $\sqrt[m]{a^p} \cdot \sqrt[n]{b^q} = \sqrt[mn]{a^{pn} \cdot b^{qm}},\ a,b \geq 0$;
14. $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}},\ a \geq 0$;
15. $\sqrt[m]{a^p} : \sqrt[n]{b^q} = \sqrt[mn]{a^{pn} : b^{qm}},\ a \geq 0, b > 0$;
16. $\sqrt{a^2} = |a|,\ a \in \mathbf{R}$;
17. $\sqrt[2n+1]{-a} = -a^{\frac{1}{2n+1}} = -\sqrt[2n+1]{a},\ a \geq 0$;
18. $\left(\sqrt[2n+1]{-a}\right)^{2n+1} = -a,\ a \geq 0$;
19. $\sqrt{a} + \sqrt{b} = \sqrt{a + b + 2\sqrt{ab}},\ a,b \geq 0$;
20. $\sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A+C}{2}} \pm \sqrt{\frac{A-C}{2}}$, dacã şi numai dacã $A^2 - B = C^2$;
21. Expresia conjugatã a lui $\sqrt{a} \pm \sqrt{b}$ este $\sqrt{a} \mp \sqrt{b}$ iar pentru $\sqrt[3]{a} \pm \sqrt[3]{b}$ este $\sqrt[3]{a^2} \mp \sqrt[3]{ab} + \sqrt[3]{b^2}$.

Modului unui numãr real

Definiție: $|x| = \begin{cases} -x, & daca\ x < 0 \\ 0, & daca\ x = 0 \\ x, & daca\ x > 0 \end{cases},\ \forall x \in \mathbf{R}$

Proprietãţi: $\forall\ x,y \in \mathbf{R}$, avem:

1. $|x| = 0 \Leftrightarrow x = 0$;
2. $|-x| = |x|$;
3. $|x| = |y| \Leftrightarrow x = y$ sau $x = -y$;
4. $|x| = a \Leftrightarrow -a = x = a, a \in$ R;
5. $-|x| \leq x \leq |x|$;
6. $|x + y| \leq |x| + |y|$;
7. $|x - y| \leq |x| + |y|$
8. $||x| - |y|| \leq |x - y|$;
9. $||x| - |y|| \leq |x + y| \leq |x| + |y|$;
10. $|xy| = |x| \cdot |y|$;
11. $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}, y \neq 0$